Universidad Tecnológica de Pereira

VII

VII TALLER DE ANÁLISIS NO LINEAL Y ECUACIONES DIFERENCIALES PARCIALES

28/29/30/31 mayo 2025

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VII TALLER DE ANÁLISIS NO LINEAL. UNIVERSIDAD TECNOLOGICA DE PEREIRA.
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Welcome

Welcome to Pereira 2025!

We are delighted to welcome friends and colleagues to our home city of Pereira for the VII WORKSHOP ON NONLINEAR ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS 2025.

Continuing the successful trend of past events, the 2025 Workshop in Pereira shows how important the VII WORKSHOP ON NONLINEAR ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS has become as a regular platform accommodating the rapid pace of progress in these fields and for presenting the results of studies which have a direct impact on the applications.

The Workshop will provide a Scientific Programme that builds on the highly successful models from previous Congresses, while incorporating innovative suggestions from valuable stakeholders.

This Congress will feature presentations of some of the most recent research, including the existence, multiplicity, and uniqueness of solutions, and a priori estimates for elliptic problems for single equations as well as for systems. We will also find conversations about semi-positone problems with the $p-$Laplacian operator. In addition, we will have some presentations on dynamical systems and fractional differential equations.

We wish you a pleasant stay in Pereira. Enjoy the Workshop!

Organizing Committee

- Carlos Ramírez (Universidad Tecnológica de Pereira)
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VII WORKSHOP ON NONLINEAR ANALYSIS AND PDEs
Universidad Tecnológica de Pereira
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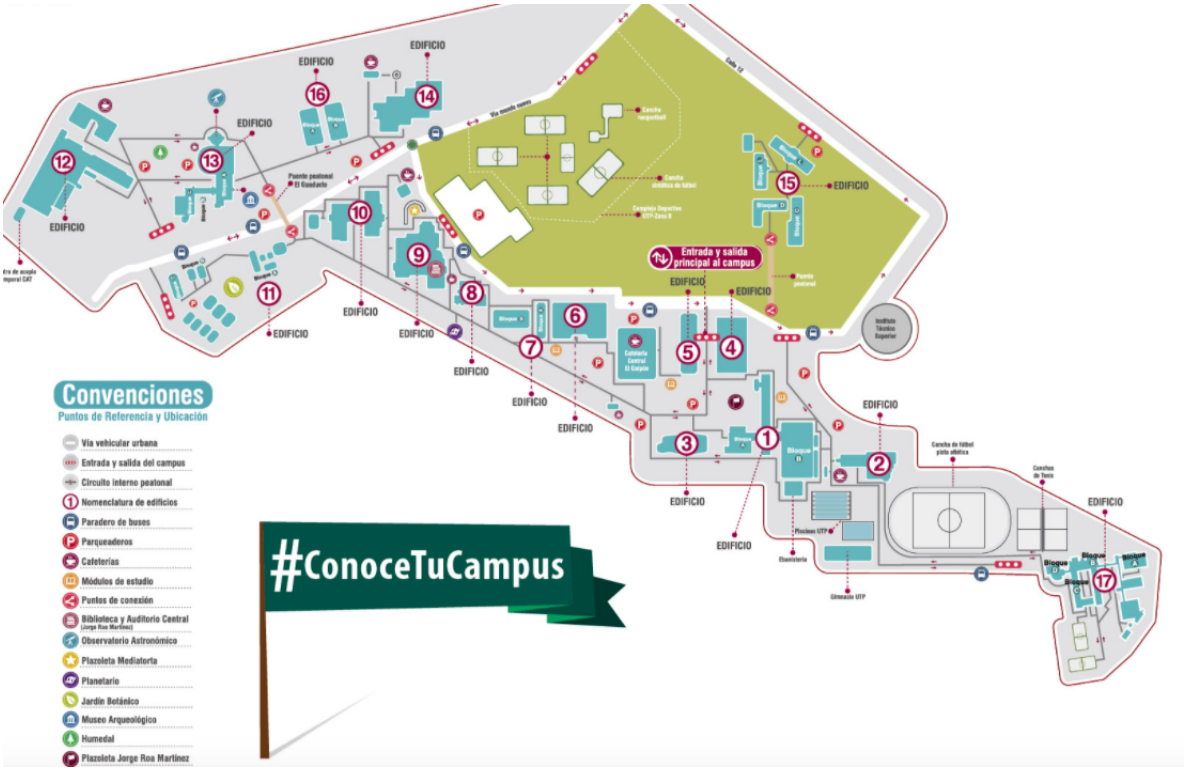

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General Information

Location

The **VII edition of the workshop** will take place in **Pereira, Colombia,** the capital of the department of Risaralda and one of the main cities in Colombia's Coffee Cultural Landscape, a UNESCO World Heritage Site.

Pereira is a vibrant and dynamic city located in the heart of the Colombian Andes. With a population of approximately 500,000 people, it serves as a key economic and cultural hub in the region. Surrounded by lush mountains and coffee plantations, Pereira offers a unique blend of urban development and natural beauty.

The city sits at an elevation of around 4,600 feet, giving it a pleasant year-round climate. Average daily temperatures range from 65°F to 80°F, with warm days and cooler nights. The weather is generally mild, and the frequent rain showers typical of the region help keep the environment green and the air fresh.

Pereira is known for its welcoming people, strong coffee tradition, and growing academic and research environment. Its central location makes it easily accessible from other major cities in the *Eje Cafetero*, such as Manizales and Armenia. The city also offers modern infrastructure, excellent cuisine, and proximity to natural attractions such as the Otún Quimbaya Flora and Fauna Sanctuary and Los Nevados National Natural Park.

Overall, Pereira provides an ideal setting for academic exchange, collaboration, and inspiration, blending scientific dialogue with the charm of Colombia's coffee region.

Useful Phone Numbers

In case of any health emergencies, please call: 123

Math department of the UTP, Pereira: (+57) (606) 313 7119

Schedule

IN CONSTRUCTION!!!

Abstracts of Talks

PLENARY LECTURES

(55 Minutes each)

Contributions of Professor José Francisco Caicedo to Nonlinear Analysis.

Alfonso Castro*
Harvey Mudd College

Abstract

A summary of the contributions of Professor José Francisco Caicedo to Nonlinear Analysis will be presented. His work focused on semilinear differential equations with discrete spectrum. From his doctoral thesis—where he studied a wave equation whose eigenvalues all have infinite multiplicity—to the doctoral theses he supervised, several results will be discussed, along with the open problems that have emerged from his research.

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Characterization of Triebel–Lizorkin Spaces via the Continuous Wavelet Transform.

Jaime Navarro Fuentes*
Universidad Autónoma Metropolitana de Mexico

Abstract

The so-called *Triebel–Lizorkin spaces* $F_p^{r,q}(\mathbb{R}^n)$ consist of all functions $f \in L^p(\mathbb{R}^n)$, with $1 < p < \infty$, that possess a certain degree of regularity determined by the parameter $r \in (0,1)$, and where the parameter $q \in (1,\infty)$ further refines this regularity. In particular, when $p = q = 2$, the space $F_2^{r,2}(\mathbb{R}^n)$ coincides with the *fractional Sobolev space* $H^r(\mathbb{R}^n)$; and when $p = q = \infty$, we have $F_\infty^{r,\infty}(\mathbb{R}^n) = C^r(\mathbb{R}^n)$, where $C^r(\mathbb{R}^n)$ denotes the *Hölder-type spaces* of order $r \in (0,1)$.

Regarding the *continuous wavelet transform* $(L_h f)(a,b)$, it arises as an alternative to classical time-frequency localization methods, where $(a,b) \in (0,\infty) \times \mathbb{R}^n$. In this context, $f \in L^2(\mathbb{R}^n)$ and the function $h \in L^2(\mathbb{R}^n)$ satisfies a certain admissibility condition.

In this talk, we will present a *characterization of Triebel–Lizorkin spaces via the continuous wavelet transform*, which allows us to study the regularity of weak solutions $u \in L^p(\mathbb{R}^n)$ to the equation $Qu = f$, where f belongs to the space $F_p^{r,q}(\mathbb{R}^n)$, with $1 < p, q < \infty$, $0 < r < 1$, and $Q = \sum_{|\beta| \le m} c_\beta \partial^\beta$ is a differential operator of order $m > 0$ with constant positive coefficients c_β.

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On the existence of solitary wave solutions for a fifth order Kaup-Kupershmidt-KdV type equation

José Raúl Quintero *
Department of Mathematics, Universidad del Valle

Abstract

In the present work, we consider the general class of evolution models, named the generalized Kaup-Kupershmidt-KdV equation, given by

$$u_t + \mu\partial_x^3 u + \alpha\partial_x^5 u + \partial_x(\gamma u_x^2 + P(u)) = 0 \tag{1}$$

where P is a polynomial and μ, α and γ are constants. This evolution model includes many well known models like the Korteweg-de Vries equation, the modified Korteweg-de Vries equation, and the Kaup-Kupershmit-Korteweg-de Vries equation. Contrary to the situation in many water wave models, the evolution model considered has no a Hamiltonian structure. Using the Fourier transform, the existence of solitary wave solutions for this model is equivalent to find a fixed point, for which we use the standard Picard method choosing appropriately the initial condition. We also include a brief discussion on the non existence of solitary wave solutions.

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A metapopulation model with almost periodic colonization and extinction rates

Pablo Amster *
Department of Mathematics, Universidad de Buenos Aires

Abstract

In this talk, we shall present an almost periodic version of a metapopulation model which generalizes the classical Levins approach by considering several species in competition and affected by habitat destruction. Persistence/extinction scenarios shall be described by means of the theory of exponential dichotomies, which shall be introduced in a self-contained manner.

This is a joint work with G. Robledo and D. Sepúlveda

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Uniqueness of positive solutions for a class of nonlinear elliptic equations with Robin boundary conditions.

Ratnasingham Shivaji*
University of North Carolina at Greensboro, USA

Abstract

We prove uniqueness of positive solutions to the BVP

$$\begin{cases} -\Delta u = \lambda f(u) \text{ in } \Omega, \\ \frac{\partial u}{\partial n} + bu = 0 \text{ on } \partial\Omega, \end{cases}$$

when the parameter λ is large independent of $b \in (0, \infty)$. Here Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $f : [0, \infty) \to [0, \infty)$ is continuous, sublinear at ∞, and satisfies a concavity-like condition for u large.

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Positive solutions for slightly subcritical quasilinear problems

Rosa Pardo*
Universidad Complutense de Madrid, Spain,

Abstract

We focus on semilinear elliptic equations involving sign-changing weight function and a nonlinearity of subcritical nature understood in a generalized sense. Using an Orlicz-Sobolev space setting, we consider superlinear nonlinearities without polynomial growth, and state sufficient conditions guaranteeing the Palais-Smale condition. We study the existence of a bifurcated branch of classical positive solutions, containing a turning point, and providing multiplicity of solutions.

We extend this result to the p- Laplacian operator, with changing sign weights. The problem has positive solutions bifurcating from the trivial solution set at the two principal eigenvalues of the corresponding linear weighted boundary value problem. Drabek's bifurcation result applies when the nonlinearity is of power growth. We extend Drabek's bifurcation result to slightly subcritical nonlinearities.

Those are joint works with Mabel Cuesta, ULCO, see [1, 2].

References

[1] M. Cuesta and R. Pardo. Positive solutions for slightly subcritical elliptic problems via Orlicz spaces. *Milan J. Math.*, 90(1):229–255, 2022.

[2] M. Cuesta and R. Pardo. Bifurcation for indefinite-weighted p-laplacian problems with slightly subcritical nonlinearity. *Mathematische Nachrichten*, 297(11):3982–4002, 2024.

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TALKS

(30 Minutes each)

Qualitative analysis for a pearl model

Alexander Gutierrez*
Universidad Tecnológica de Pereira

Abstract

We establish necessary conditions for the existence of periodic and subharmonic solutions of a differential equation that describes the motion of a pearl on a circular ring rotating with a constant angular velocity ω and subject to a periodic forcing. Our approach involves using the method of sub- and supersolutions and some techniques presented by Zanolin–Boscaggin and Ureña.

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A review on the range of the semilinear wave equation with non-monotone nonlinearity

Arturo Sanjuán*
Universidad Distrital Francisco José de Caldas

Abstract

We review recent results led by professor Alfonso Castro on the existence of weak solutions to the semilinear wave equation with non-mononote nonlinearity. We assume periodic-boundary conditions and the forcing term in L^2.

References

[1] Caicedo, J. F., Castro, A., Duque, R., & Sanjuan, A. (2023). A semilinear wave equation with non-monotone nonlinearity. *Electronic Journal of Differential Equations, Special Issue 02, 81-86.*

[2] Caicedo, J. F., Castro, A., & Duque, R. (2021). A semilinear wave equa- tion with non-monotone nonlinearity and forcing flat on characteristics. *Electronic Journal of Differential Equations, Special Issue 01, 91-99.*

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Some aspects of positive solutions for Hamiltonian elliptic systems with concave-convex nonlinearities

Carlos Vélez*
Universidad Nacional de Colombia Sede Medellín

Abstract

In this talk, we study aspects related to the existence of multiple solutions for the boundary value problem:

$$\begin{cases} -\Delta u = \lambda v^r + v^p & \text{in } \Omega, \\ -\Delta v = \mu u^s + u^q & \text{in } \Omega, \\ u > 0,\, v > 0 & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, Δ is the Laplace operator, $\mu, \lambda \geq 0$, $r, s \in (0,1)$, and $p, q \in (1, \infty)$.

The results presented here are motivated by the work of Ambrosetti, Brezis, and Cerami (see [1]), and establish that under suitable conditions, there exists a continuous and decreasing function $\lambda_* : [0,\infty) \to [0,\infty)$ such that in the region

$$S = \left\{(\lambda, \mu) \in \mathbb{R}^2 : 0 \leq \lambda \leq \lambda_*(\mu)\right\},$$

the system (P) has at least one solution. In contrast, in the region outside S, the system (P) has no solutions, while in the interior of S, the system (P) has at least two solutions.

These aspects are part of a work in progress in collaboration with Oscar Agudelo (University of West Bohemia) and Bernhard Ruf (University of Milan).

References

[1] A. Ambrosetti, H. Brezis, and G. Cerami, Combined effects of concave and convex nonlinearities in some elliptic problems, *Journal of Functional Analysis*, 122(2):519–543, 1994.

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About some generalized nonlocal diffusion models.

Cesar A. Gómez S.*
Universidad Nacional de Colombia Sede Bogotá

Abstract

One of the classical models of nonlocal diffusion is given by

$$\begin{cases} u_t(x,t) = \int_\Omega J(x,y)(u(y,t) - u(x,t))\, dy \\ \\ u(x,0) = u_0(x), \end{cases}$$

where Ω is a connected open subset of $\mathbb{R}^N$, and $J(x,y)$ and $G(x,y)$ are kernels that determine the jump probability in the problem.

We present some generalizations of this problem, which include generalized boundary conditions, among which we consider Neumann-type and Dirichlet-type boundaries. Some results related to the existence and uniqueness of solutions, as well as a comparison principle, are presented.

This work is in collaboration with Cristian Minia, a Master's student in Mathematics at UNAL Bogotá.

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Waves in fluids: generation and propagation.

David Andrade*
Universidad del Rosario

Abstract

In this work, we present a new formulation of the Euler equations for an ideal fluid, bounded above by a free surface representing the ocean surface, and below by a moving seabed. The purpose of these equations is to model the waves generated at the surface by seabed deformations. Additionally, we will present a numerical method that allows us to simulate the solutions of the PDE system and thus study the waves generated in this manner.

All details and results of this work can be found in [1]. At the time of writing this abstract, the article has been accepted for publication in the *SIAM Journal on Applied Mathematics*.

References

[1] J. V. P. Poletto, D. Andrade, M. V. Flamarion, and R. Ribeiro-Jr, Full euler equations for waves generated by vertical seabed displacements, *arXiv preprint arXiv:2310.10596, (2023).*

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Applications of a sub-super solution theorem to elliptic problems

Diana M. Sanchéz*
Universidad Nacional de Colombia sede Manizalesr

Abstract

The purpose of the talk is to apply the sub-super solution theorem introduced by Eun Kyoung Lee, R. Shivaji and Jinglong Ye in their paper "Classes of infinite semipositone systems" to two singular elliptic problems involving a p-Schrödinger type operator with homogeneous Dirichlet condition.

This research was conducted in collaboration with Professors M. Chhetri and R. Shivaji.

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An Example on the Quadrature Method

Diego Alexander Castro Guevara*
Universidad Tecnológica de Pereira
Universidad Nacional de Colombia – Manizales, PhD (Student)

Abstract

In this presentation, we will show the existence, uniqueness, positivity, and symmetry of the solution to the boundary value problem for the semilinear ordinary differential equation

$$u''(t) + u(t)^{2n} = 0; \quad u(-1) = u(1) = 0$$

where $n \in \mathbb{Z}^+$. The ideas behind the proof are mainly based on the well-known quadrature method and the maximum principle. Finally, for the case $n = 1$, we will numerically illustrate the theoretical results and the geometry of the solution.

References

[1] José F. Caicedo, Alfonso Castro; REcuaciones semilineales con espectro discreto; *Universidad Nacional de Colombia-Sede Bogotá.*

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Existence and nonexistence of positive solutions for a class of quasilinear problems.

Emer Lopera*
Universidad Nacional de Colombia Sede Manizales

Abstract

In this talk we explore a class of problems of the type $-\Delta_p(u) + V(x)|u|^{p-2}u = \lambda f(u)$, in Ω, $u|_{\partial\Omega} = 0$, where Δ_p stands for the p-Laplacian operator, $p > 1$, and $\Omega \subseteq \mathbb{R}^N, N > 1$, is a bounded domain with smooth boundary. Here λ is a real number, $f : [0, \infty) \to \mathbb{R}$ is continuous function (the reaction term) and $V : \Omega \to \mathbb{R}$ (the potential) belongs to $L^\infty(\Omega)$. Among other hypotheses that shall be specified in the talk, the reaction term is super-lineal and sub-critical at infinity. The interaction of the potential with the first eigenvalue of the problem $-\Delta_p u = \lambda|u|^{p-2}u$ in Ω, $u|_{\partial\Omega}\, u0$, shall be crucial in our analysis. We establish existence and multiplicity of positive solutions for any $\lambda > 0$ sufficiently small. We also establish the nonexistence of positive solutions whenever λ is sufficiently large.

Joint work with: M., Chhetri & R., Shivaji.

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On the existence, regularity and control theory of some chemotaxis model

Élder J. Villamizar-Roa*
Universidad Industrial de Santander

Abstract

In this talk we will present some recent results on the existence and control theory related to the following three-dimensional parabolic-parabolic chemo-repulsion model with nonlinear signal production and bilinear control, including or not a logistic reaction term:

$$\begin{cases} \partial_t u - \Delta u = \nabla \cdot (u \nabla v) + ru - \mu u^p \\ \partial_t v - \Delta v + v = u^p + f v 1_{\Omega_c} \end{cases}$$

where $1 \leq p < +\infty, r, \mu \geq 0$, and $f = f(t,x)$ is the control function acting on a subdomain $Q_c = (0,T) \times \Omega_c \subset Q = (0,T) \times \Omega$. This system is endowed with initial and non-flux boundary conditions. We prove the existence of global weak solutions of this controlled problem when $f \in L^{5/2}(Q_c)$, analyzing the role of the diffusion and the logistic terms to get energy estimates. We prove the existence of global optimal solutions and derive first-order necessary optimality conditions for local optimal solutions.

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Exponential decay rate of energy for the BBM-KP model under localized damping in $\mathbb{R}^2$

Fernando A. Gallego*
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Abstract

In this talk, we discuss recent results concerning the **energy decay properties of the BBM-KP equations** (types I and II) posed on the whole plane $\mathbb{R}^2$, in the presence of a **localized damping** mechanism. The model under consideration is given by:

$$\begin{cases} u_t + u_x + u^p u_x - u_{xxt} + \gamma \partial_x^{-1} u_{yy} + a(x,y)u = 0, & (x,y) \in \mathbb{R}^2,\ t > 0, \\ u(x,y,0) = u_0(x,y), & (x,y) \in \mathbb{R}^2, \end{cases}$$

where $\gamma = \pm 1$ and $p \geq 1$ is an integer. This system can be seen as a *regularized alternative to the Kadomtsev–Petviashvili (KP) equations*, much like the BBM (Benjamin–Bona–Mahony) equation serves as a regularization of the classical KdV equation.

We prove that the **energy associated with the Cauchy problem decays exponentially** when the damping term $a(x,y)$ is nontrivial in a suitable subregion of the domain. To complement the theoretical analysis, we present **numerical simulations** based on a spectral finite difference scheme, which illustrate and support the exponential stabilization of solutions under localized damping.

This is a joint work with Victor Hugo Gonzalez Martínez (UFPE, Brazil) and Juan Carlos Muñoz Grajales (UNIVALLE, Colombia).

References

[1] F.A. Gallego, V.H. Gonzalez-Martínez and J.C Muñoz-Grajales. Global Stabilization for the BBM-KP equations on $\mathbb{R}^2$ *https://arxiv.org/pdf/2410.01998*

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Existence of Multiple Solutions for a Quasilinear Elliptic Problem

Jorge Cossio *
Universidad Nacional de Colombia Sede Medellín

Abstract

The talk will be concerned with the existence of solutions for the quasilinear elliptic boundary value problem

$$\begin{cases} \Delta_p u + f(u) &= 0 \quad \text{in} \quad \Omega, \\ u &= 0 \quad \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplace operator, $p > 1$, $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ is a bounded and smooth domain, and $f : \mathbb{R} \to \mathbb{R}$ is a nonlinear function such that $f(0) = 0$.

We prove the existence of multiple solutions for problem (1) when the p-derivative at zero and the p-derivative at infinity of the nonlinearity f are greater than the first eigenvalue of the p-Laplace operator. This result extends to quasilinear equations a theorem due to J. Cossio, S. Herrón, and C. Vélez [2] for the semilinear case. Our proof uses bifurcation from infinity and bifurcation from zero to prove the existence of unbounded branches of positive solutions (resp. of negative solutions). We show the existence of multiple solutions and we provide qualitative properties of these solutions.

This is a Joint work with S. Herrón and C. Vélez (Universidad Nacional de Colombia)

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ELusternik-Schnirelman and Morse Theory for the Van der Waals-Cahn-Hilliard equation with volume constraint

Luis Eduardo Osorio *
Universidad Tecnológica de Pereira, UTP

Abstract

We give a multiplicity result for solutions of the Van der Waals-Cahn-Hilliard two-phase transition equation under volume constraints on a closed Riemannian manifold. In its vari- ational formulation, the Allen-Cahn equation

$$\epsilon\Delta u - \frac{1}{\epsilon}W'(u) = 0,$$

arises as the Euler-Lagrange equation of the energy functional

$$E_\epsilon(u) = \epsilon\int_M \frac{1}{2}|\nabla u|^2 dV + \frac{1}{\epsilon}\int_M W(u)dV.$$

Historically, this functional has been used to model phase transitions -from liquid?vapor phenomena to binary alloy mixtures and ferromagnetic behavior- and its ubiquity is evident in various applications. Our proof combines classical Lusternik-Schnirelmann and Morse theory with the pho- tography method. This technique associates to each point of the manifold a bell-shaped function (an ϵ-approximation of a geodesic ball with prescribed volume) and recovers the point through a barycenter map that identifies where most of the function's mass con- centrates. Recent advances in Riemannian isoperimetry for small volumes underpin this approach. It is important to note that, although the central multiplicity result remains valid, a mistake was discovered in the original formulation regarding the photography map. The earlier claim that the photography map takes values in energy sublevels arbitrarily close to the infimum is incorrect due to the dependence of the energy on the local scalar curvature. This corrigendum provides a revised description of the photography map's image and an alternative construction of the barycenter map, thereby ensuring that the main theorem holds.

This is a joint work with Vieri Benci (Univ. Pisa), Stefano Nardulli (CMCC-UFABC), Paolo Piccione (IME-USP) and Dario Corona (IME-USP)

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Gradient systems of polynomial solutions to the Laplace equation

Oscar Ramírez*
Facultad de Ciencias Matematicas y Naturales,
Universidad Distrital Francisco José de Caldas, Bogota, Colombia

Abstract

In this work, we study the configuration of orbits in gradient systems associated with polynomial solutions of the Laplace equation in $\mathbb{R}^2$. In particular, we characterize all local phase portraits at finite and infinite singular points of these systems and classify the topologically non-equivalent phase portraits in the Poincaré disk for polynomials of degree less than five.

This is joint work with Jaume Llibre (Universitat Autónoma de Barcelona).

References

[1] J. Llibre and O. Ramírez, Gradient systems of harmonic polynomials, *Journal of Differential Equations* **269** *(2020), no. 11, 10073-10084.*

[2] J. Llibre, Integrability of polynomial differential systems, *Handbook of Differential Equations, Ordinary Differential Equations, Eds. A. Cañada, P. Drabek and A. Fonda, Elsevier, 2004, pp. 437-533.*

[3] J. Llibre and R. Oliveira, Quadratic systems with invariant straight lines of total multiplicity twc having Darboux invariants, *Commun. Contemp. Math.* **17**(2015), 1450018, 17pp.

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Existence of weak solutions for the Burgers equation with time-dependent coefficient

Oscar Fernando Rojas Matamoros*
Escuela de Matemáticas y Estadística,
Universidad Pedagógica y Tecnolpógica de Colombia

Abstract

In this talk, we present results on the existence of solutions for the Burgers equation with a time-dependent coefficient,

$$u_t + \alpha(t)uu_x = 0,$$

where $\alpha \in C([0,\infty))$ and $\alpha(t) > 0$ for each $t \geq 0$. The Burgers equation has been extensively studied in various contexts [1, 2, 3, 4, 5, 6]. In 2020, Wang, Zhang, Li, and Xin [7] used a generalization of the Hopf–Cole transform to obtain solutions of the Burgers equation with time-dependent coefficients and linear damping. More recently, De la Cruz, Lu, and Wang [6] found solutions for the Riemann problem for the Burgers equation with time-dependent coefficients and linear damping. When the linear damping tends to zero, they obtain weak solutions for the Riemann problem for the Burgers equation with a time-dependent coefficient.

In this talk, we present solutions for the Cauchy problem for the Burgers equation with a time-dependent coefficient, where the initial data satisfies suitable growth conditions.

References

[1] J.M. Burgers, Mathematical examples illustrating relations occurring in the theory of turbulent fluid motion. *Verhandelingen der Koninklijke Ne- derlandsche Akademie van Wetenschappen, Afdeeling Natuurkunde (1st Sect.) 17(2) (1939) 1-53.*

[2] J.M. Burgers, Application of a model system to illustrate some points of the statistical theory of free turbulence. *Proc Acad Sci Amsterdam 43 (1940) 2-12*

[3] J.M. Burgers, A mathematical model illustrating the theory of turbulence. *Advances in Applied Mechanics 1 (1948) 171-199.*

[4] E. Hopf, The partial differential equation $u_t + uu_x = \varepsilon u_{xx}$. *Commun Pure Appl Math 3(3) (1950) 201-230.*

[5] J.D. Cole, On a quasi-linear parabolic equation occurring in aerodynamics. *Quart Appl Math 9 (1951) 225-236.*

[6] R. De la cruz, Y. Lu, X. wan, QRiemann problem for a general variable coefficient Burgers equation with time-dependent damping, *International Journal of Non-Linear Mechanics 162, (2024) 104703.*

[7] M. Wang, J. Zhang, E. Li, X. Xin, The generalized Cole-Hopf transforma- tion to a general variable coefficient Burgers equation with linear damping term. *Applied Mathematics Letters 105 (2020) 106299.*

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Riemann Problems for hyperbolic systems of conservation laws with a time-dependent Coulomb-like friction term

Richard De la cruz*
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Universidad Pedagógica y Tecnológica de Colombia

Abstract

In this talk, we present recent results on the construction of Riemann solutions for a class of strictly and nonstrictly hyperbolic, nonlinear systems of conservation laws influenced by a time-dependent Coulomb-like friction term. The analysis focuses on the Riemann problem, where we describe the structure of both elementary and non-elementary waves. In particular, we obtain contact discontinuities and delta shock waves. These results contribute to the understanding of singular wave phenomena in systems with time-dependent coefficients in the source terms.

This is joint work with Wladimir Neves (Universidade Federal do Rio de Janeiro - UFRJ, Brazil).

References

[1] R. De la cruz and W. Neves,. Riemann problem for a nonsymmetric Keyfitz-Kranzer and pressureless gas systems with a time-dependent Coulomb-like friction term, *Nonlinear Analysis: Real World Applications* 84, 104301(2025)

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The Laplacian operator and generalizations

Sigifredo Herrón*
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Abstract

This talk covers some elements of the Laplacian differential operator. Emphasis will be given to the most important results in the context of functional analysis and elliptic partial differential equations involving it. Then, some of the most important generalizations of this operator, the similarities and differences with the Laplacian, and the known results in the context of functional and elliptic partial differential equations will be presented.

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Mathematical Model and Real-Time Sensor for the Dynamics of Dissolved Oxygen and Temperature in Closed Fish Transport Systems

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Abstract

The transport of fish in confined systems poses significant physiological and logistical challenges due to the dynamics of dissolved oxygen (DO) in the water. DO concentration is a critical parameter that determines the survival and well-being of aquatic organisms, as it directly affects their metabolism and their ability to adapt to stressful conditions. This study develops and analyzes a mathematical model based on ordinary differential equations (ODEs) that describes the temporal evolution of DO in transport tanks, integrating physical, biological, and environmental variables.

The proposed model is part of the TRAPES Project (Transport of Fish in Closed Systems), which introduces an innovative approach by considering oscillatory dynamics induced by critical DO thresholds. This allows for a better understanding of oxygen consumption and recovery regimes during transport.

This is joint work with Luis Eduardo Bermúdez (Universidad Surcolombiana).

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